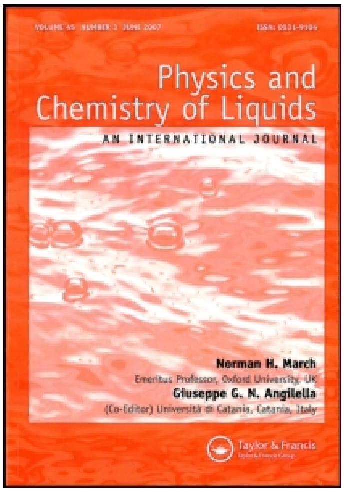

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DUAL RELATIVITY AND LORENTZ TRANSFORMATIONS FOR LIQUID-LIKE MEDIA AND STATISTICS FROM SCALING CONCEPTS FOR GEOMETRY

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A dual class of Lorentz transformations (dual LT) for linear and Brownian motions in liquid-like media, is presented and discussed. It descends from two LT groups, for self-diffusion in simple liquids (BLT) and its geometrical analogy (GLT), which turned out to represent promising basic tools to deal with statistics at different length scales. Time dilation and length contraction of BLT occur upon ordering the originally indistinguishable molecular disorder in the liquid medium (i.e., a diffusivity lowering), giving rise to universality and scaling laws in polymer solutions. Dual LT exhibit a rich phenomenology, leading to a scale-dependent motion concept, where Brownian and quantum movements somewhat correspond. Density correlations in simple liquids, for instance, are suggested on this basis to behave like radial wave functions at the atomic scale. We also report some remarks on statistics in general, and its connection towards geometry.

Keywords: Dual relativity; Geometrical scaling law; Statistical physics; Polymer solution; Simple liquid

INTRODUCTION

A relativistic theory of Brownian movement for self-diffusion in liquids (BR) has been recently formulated to derive the main scaling laws of polymer solutions [1–3]. In the Einstein theory of relativity [4], any couple of standard metric tensors, g and g', leave the space time interval (s) unchanged upon any coordinate change, like $x \to x'$:

$$ds'^2 = ds^2 = -g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{1}$$

and transform as:

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta} \tag{2}$$

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In BR instead, let time $t \equiv z^0$ and diffusivity be $D = -\bar{g}_{00}$, the new Brownian interval:

$$d\sigma^2 = -\bar{g}_{\mu\nu} dz^\mu dz^\nu \tag{3}$$

resorts to mean square displacements, for example according to $dx^\mu \rightarrow \sqrt{\overline{dx^\mu}}$ and $dt \rightarrow \sqrt{dt}$. They implicitly correspond to given probability distributions, and so (polymer) statistics may be translated in terms of geometry. It was afterwards suggested, precisely, that *scaling laws of polymer solutions arise from ordering a space indistinguishably disordered* [5].

BR points first out a novel definition of macromolecule in liquid. It would be identified by the curvature change caused by a local mobility variation produced into diffusive spacetime. The simplest example is evidently provided by the Brownian special relativity (BSR) *versus* random walk, where lengths are contracted upon lowering the randomness of the host medium. It suffices starting from:

$$d\sigma^2 = D\,dt - (\overline{dx^2} + \overline{dy^2} + \overline{dz^2}) \tag{4}$$

and, owing to a transformation like $1 \rightarrow N$, to locally decrease the suspension mobility.

We originally considered N liquid molecules collected in a linear tube of length Na, a being the molecular size. When these units are merged together into a single chain (and chemical transformations are absent) the Lorentz transformations (LT) provide a geometrical constraint ruling the transition between rod-like and curvilinear tubes, i.e., from single molecules to a corresponding macromolecule in solution. Letting the new Lorentz factor be $1 - \Delta D/D = D_N/D = \delta$ [1], the Brownian LT taking place are (in unit of D, see Appendix I):

$$\begin{cases} t' = (2-\delta)t + (1-\delta)\overline{r^2} \\ \overline{r'^2} = \delta\overline{r^2} + (\delta - 1)t \end{cases} \tag{5}$$

and, inversely ($\delta \rightarrow \delta^{-1}$):

$$\begin{cases} t = (2-\delta^{-1})t' + \dfrac{\delta-1}{\delta}\overline{r'^2} \\ \overline{r^2} = \delta^{-1}\overline{r'^2} + \dfrac{1-\delta}{\delta}t \end{cases} \tag{6}$$

They result into the following laws of time dilation and length contraction (TDLC):

$$\begin{cases} dt' = \delta^{-1}\,dt \\ d\overline{r'^2} = \delta\,d\overline{r^2} \end{cases} \tag{7}$$

which return random walk size and Rouse time behaviors of ideal phantom chains, i.e., without excluded volumes (or self-avoiding walks) [6]. For Rouse-like coils, in fact, one has $\delta \sim N^{-1}$ and thus:

$$\begin{cases} \rho' = \sqrt{\delta}Na = \sqrt{N}a \\ t' \sim \dfrac{Nt_1}{\delta} = N^2 t_1 \end{cases} \tag{8}$$

primes still denoting the polymer frame, t_1 being an intrinsic molecular time and $\rho \equiv \overline{r^2}$. Another manner of focusing the results achieved is starting from $\rho^2 t = \text{const}$ (TDLC, in Eq. 7) and considering the universal exponents associated ($\nu = \ln \rho' / \ln N$, $\tau = \ln t' / \ln N$):

$$\begin{cases} \nu = 1\,, & \tau = 1 \quad \text{(rod-like tube)} \\ \nu = \dfrac{1}{2}\,, & \tau = 2 \quad \text{(ideal coil)} \\ \nu = \dfrac{3}{5}\,, & \tau = \dfrac{9}{5} \quad \text{(real coil)} \end{cases} \tag{9}$$

They may only be determined *a priori* in the rod-like case, setting the constant value to $2\nu + \tau = 3$. While the Rouse scaling (second row) was surely expected being part of this behavior, we can surprisingly ascribe to the previous table the swollen Flory coil as well. It is endowed with mass and would correspond to $\delta \sim N^{-4/5}$, but there is no reason at the moment to suppose a mobility scaling the like this in the BL factor.

In general, every metric information is primarily summarized by the diffusion coefficient behavior, $D = D(z^k, t)$. This is what we started from, in examining real coil and polymer melts. In the first case, the excluded volume was modeled through a Schwarzschild singularity in four dimensions, and an analogy between black holes and single real chains in unmaterial media was implicitly pointed out. Diffusivity was assumed to vanish at the (Schwarzschild) coil radius, identifying the average end-to-end dimension of a long molecule lying into a curved but empty space. General relativity (GR), in the second instance, yielded a new scaling behavior in the form of a (weakly) covariant equation, denoting a modified Stokes law holding in fluids highly concentrated. Note that, while BSR makes use of a two-dimensional metric, averaged over the root mean square that the random motion depicts, line elements of GR involve instead all spatial coordinates (still with $d\tau = \sqrt{dt}$ and $dz = \sqrt{\overline{dx^2}}$):

$$d\sigma^2 = \overline{g}_{00} d\tau^2 - \overline{g}_{\rho\rho} d\rho^2 - \overline{g}_{\theta\theta} d\theta^2 - \overline{g}_{\varphi\varphi} d\varphi^2 \tag{10}$$

The framework above might be termed as weakly covariant. Clearly, liquid diffusivities cannot be regarded as impassible limits of nature, like Einstein could state for the light speed. One would rather expect here a limit horizon, able to reorganize itself and interact with the same physical world which it bounds. Secondly, the Brownian LT are not invertible and the ordinary principle of covariance holds no longer. This may sound as physically uncorrect, but is simply a formal consequence of using (Brownian) LT to model a statistical phenomenon. On the one hand, similarly to SR, in the simple liquid frame the proper polymer length Na is seen to contract by $\sqrt{N}$, whereas time appears to run slow by N^2. Such properties find their origin into local geometry transformations and do not belong to the chain as a whole [6]. On the other, however, BLT fix definitely the relationship between any couple of length or time intervals, and no discrepancy between observers at rest and in relative diffusion can take place. Translating into another insight, there is no possible equivalence for two different disorder degrees. Whenever a frame of reference detects some time dilation and length contraction, it is allowed to conclude being linked to the highest disorder (the larger diffusivity) and thus, surprisingly, being in absolute (Brownian) motion with respect to the other observer. In short, *statistics does not admit a formulation that is motion and/or scale covariant* [5]. The either concept in BSR, which follows accordingly, identifies a diffusive Brownian medium that, on average, is still insensitive to translational movements (see SR). It would be characterized by vanishing *mean* velocity and drift, but provided with non-null diffusivity and velocity fluctuations. There could be, in other words, another phenomenology level, interacting with the relativity paradigm, which the birth of statistics would be ascribed to.

The previous view pointed out also an interesting geometrical analogy, which has promising prospects in studying the nature of statistics in general [2,5,7]. Reformulating the picture above in terms of simple geometry (GSR) may return in fact geometrical constraints that, in principle, can hold at any length scale. We so applied LT to a scalar shape (ϕ), which localizes a measure (x) in some extended space (ϱ), for instance according to:

$$dx = \phi \, d\varrho \tag{11}$$

and, in the end, the relation achieved consisted of a simultaneous extension and shape contraction, or [7]:

$$\phi_\Lambda(\varrho') \sim \phi_\lambda(\varrho) \tag{12}$$

This is likely the simplest form of scaling laws for geometry (or geometrical scaling laws, GSL). Whenever shape is correctly regarded like some probability (or density), it can join statistics defined at two different scales, λ and Λ (see Fig. 1a). In order to complete the picture formulated, we were led then to thinking of the initial elementary like an uncertainty relation for geometry (GU): *measuring (a length) undetermines extension and shape* [5,7]:

$$\varrho\phi \sim x \tag{13}$$

Briefly, GLT come from regarding a molecular mobility decrease as equivalent to a shape that bounds some extended space portion. When we pass from diffusional

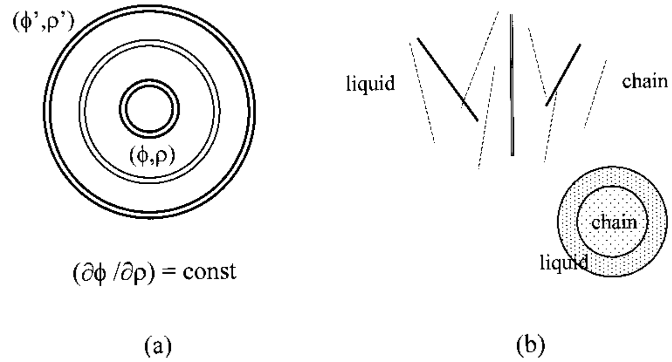

FIGURE 1 Scheme of (a) geometrical scaling (GSL) and (b) geometrical uncertainty (GU) in the liquid-chain system.

to geometrical frameworks, the condition $\delta \rightarrow \delta^{-1}$ simply transforms into $\varrho \rightarrow -\varrho$, and the first SR postulate of covariance is recovered. BLT in fact, conserve any spacetime interval, but are not invertible. Let $L_{\pm}$ be direct and inverse transformations respectively, their product differs from the identity matrix (I) by (Eqs. (5) and (6)):

$$D^{*} = L - I = \begin{pmatrix} 1 & -D \\ D^{-1} & -1 \end{pmatrix} \Delta \tag{14}$$

with $L = L_{+}L_{-} = L_{-}L_{+}$ and $\Delta = (1-\delta)^{4}/\delta^{2}$. Thus, let $z \equiv (\varrho, t)$, requiring BLT to be invertible equals to set $D^{*}z = 0$, which easily reduces to:

$$\Delta\sigma^{2}(\delta) = 0 \tag{15}$$

As expected, the former condition is satisfied by δ=1, i.e., a homogeneous system like the starting simple liquid. If $\delta \neq 1$, Δ should be conceived like some functional operator for the four-dimensional interval [8].

Pictorially, one might also imagine a lens (or else, a mirror) splitting chain and liquid frames of reference in two distinct worlds. Any length and time intervals, observed from both mirror sides, conserve their absolute values, but not their coordinates, which are indetermined (Fig. 1b). We were so brought to define an uncertainly relation for geometry, though there is a further, much simpler evidence that would encourage to introduce a principle like GU. As for quantum positions and momenta, which cannot be simultaneously determined with arbitrary precision [9], shape cannot be described independently of the extension where it lies (and viceversa). In light of GU, the same metric tensor determinant, $g = \det g_{\mu\nu}$, would express this geometrical uncertainty too, as discussed elsewhere [5]. The origin of experimental errors can be ascribed to GU as well: when a space portion can be regarded as much larger than any measure of length performable therein, then the uncertainties, in principle, may be arbitrarily reduced. At the atomic scale, extension and measure get the same order of magnitude, and the Heisemberg relation holds. So, features like the nature of the observer, even the unit of measure or, better, the scale or focus, would take a crucial role in physics. This is what another program, termed as scale relativity, investigated over the last decades as well [10]. Nevertheless, in spite of rising questions sometimes similar to those here presented, its fundamentals, and aims and results are significantly different.

To further summarize the previous picture, to pass from studying polymer statistics (BR) to statistics in general, suggested us to set a problem merely geometrical, out of time and mass concepts. BLT were interpreted in terms of their geometrical counterparts, which are now covariant (GLT), but provided with an uncertainty relation for geometry (GU). The meaning of GSL gets thereby two-fold: they link geometrically different spatial statistics and, at the same time, split up single phenomena into distinct statistical dualities depending on length scale (i.e., atom–molecules, monomer–polymer, and so on). In other words, *statistics is arising from, scaling with and indetermined by the length scale of observation* [5].

Note that, not to weaken or break covariance, it was obviously sufficient to have a linear diffusive law, like $\sqrt{\varrho} \propto t$. Of course, if disorder could evolve accordingly, BSR would formally get back to SR, but in this way statistics of polymer solution could never take place (the new Lorentz factor getting independent of large N values, i.e.,

$\delta \to 1-(1-\delta)^2 \simeq 1$). This is just the transient solution of the Langevin equation, holding at short time scales, when the frictional force has not produced yet appreciable effects [11]. Polymer statistics would so appear being intimately related to the time-dependence of the (diffusive) motion equation.

In this article, the picture above is continued towards a two-fold direction: (i) proceeding the work in progress, on macromolecules in solution and statistics in general, with attempting some more general transformation class; (ii) suggesting a heuristic derivation of distribution functions in liquids that relies on scaling concepts for geometry. It should be remarked that this preliminary study belongs to a much wider picture [5], which is waiting for results from several other investigations before to reach the novel organic view on statistical and thermal physics that we set out to formulate.

DUAL RELATIVITY AND LORENTZ TRANSFORMATIONS

(i) Dual SR

The present section derives a dual transformation class that accounts for kinematics (order) and diffusion (disorder) through a unified picture of motion. To this end, define a time τ, a radial coordinate ϱ and a limit velocity ℓ so to have, for instance, the superposition that follows:

$$\begin{cases} \ell^2\tau^2 = \kappa^2 t^2 + D_c t \\ \varrho^2 = r^2 + \overline{r^2} \end{cases} \tag{16}$$

where κ and D_c are respectively the limit speed and diffusivity. The latter value, referring to:

$$D_c = \kappa x_c \tag{17}$$

undetermines the translational motion by the limit measure x_c (also, the impassible limit measure in GSR [7]). The spacetime interval to handle now is:

$$ds^2 = \ell^2 d\tau^2 - d\varrho^2 \tag{18}$$

and therefore, letting $\varepsilon = d\varrho/d\tau$ and $\gamma = \varepsilon/\ell$, (direct) dual LT are:

$$\begin{cases} \tau' = (1-\gamma^2)^{-1/2}(\tau - \frac{\gamma}{\ell}\varrho) \\ \varrho' = (1-\gamma^2)^{1/2}(\varrho - \varepsilon\tau) \end{cases} \tag{19}$$

whilst TDLC laws become:

$$\begin{cases} d\tau = (1-\gamma^2)^{-1/2} d\tau' \\ d\varrho = (1-\gamma^2)^{1/2} d\varrho' \end{cases} \tag{20}$$

Expliciting γ provides a time-dependent relationship:

$$\gamma^2 = \frac{1}{\ell^2}\left(\frac{d\varrho}{d\tau}\right)^2_{\ell} = \frac{v^2 t + \chi D_c}{\kappa^2 t + D_c} \to \begin{cases} \beta^2 & \text{(SR)} \\ \chi & \text{(BSR)} \end{cases} \tag{21}$$

where v and χ measures the relative speed and diffusivity. Working in unit of κ ($\beta = v$) and recalling $\chi = 1 - \delta$ (see also Appendix I) carry to:

$$\begin{cases} (1 - v^2)t^2 + \delta x_c t = t'^2 + x_c t' \\ \{(1 - v^2)t + \delta x_c\}\varrho'^2 = (t + x_c)\varrho^2 \end{cases} \tag{22}$$

giving SR and BSR as special cases and pointing two distinct, elongational and time-like, Lorentz factors out:

$$\begin{cases} dt = (1 - \gamma_T^2)^{-1} dt' \\ d\varrho^2 = (1 - \gamma_L^2) d\varrho'^2 \end{cases} \tag{23}$$

which fulfill:

$$(1 - \gamma_T^2)(t' + x_c) = (1 - \gamma_L^2)(t + x_c) = (1 - v^2)t + \delta x_c \tag{24}$$

Mathematical analysis of Eq. (12) is not the aim of the present work. However, some meaningful properties can be found in Appendix II.

(ii) Particle Energy

Consider the relativistic energy of a particle with rest mass m_0 [4]:

$$E = \frac{m_0 \ell^2}{(1 - \gamma^2)^{1/2}} \tag{25}$$

and its double MacLaurin expansion in v and δ. It turns out (see Appendix III):

$$E \simeq \ell^2\left(1 + \frac{x_c}{t}\right)^{-1/2}\left(E_0 + \frac{m_0 v^2}{2} - \frac{m_0 x}{2t}\right) \tag{26}$$

where $E_0 = m_0$ is the rest energy, the second addendum denotes the kinetic term and $x = D$ stands the diffusivity contribution for. Although the expansion performed clearly affects the time behavior, SR is expectedly found again upon $t \to \infty$ and $\ell \to \kappa$.

RESULTS AND DISCUSSION

Ordered and disordered motions have been joint by means of the dual frame-work unifying SR and BSR, where ℓ specifies the limit velocity of a particle that can move either translationally or diffusionally. As expected, Eq. (21) and the analysis reported in Appendix II show that the first stage of motion is governed by BSR, tending asymptotically to get SRlike (see Fig. 2). Dual LT phenomenology is rather rich. For instance, (i) as TDLC depend on time and are only defined upon $v^2 t + (1-\delta)x_c \geq t - t'$, opposite phenomena of time contraction and length dilation can take place; (ii) time-reversal transformations like $t \to -x_c$, with $1 - v^2 \neq \delta$, produce infinite length dilation ($\varrho \to \infty$) and time contraction ($t \to 0$); (iii) as a consequence of adding the Brownian disorder to the original LT, t gets admitting two algebraical solutions. This brings us to recall the equivalence principle for entropy and matter, suggesting already the existence of a second (kinematic) time, just driven by irreversibility and disorder [12]; (iv) lastly, energy is relativistically corrected by the new limit quantity ($m_0\ell^2$) and two time-dependent diffusive contributions, negligible at large time scales.

To go further, consider the value taken by the diffusion coefficient of a particle Brownianly moving in the liquid phase. It can be expressed by the thermal Boltzmann energy, multiplied by a characteristic time and divided by its mass, $D = (\tau k_B T/m_0)$ [13]. Thus, letting $E = (1 + (x_c/t))^{1/2}\ell^2\tilde{E}$:

$$\tilde{E} \simeq E_0 + \frac{m_0\overline{v^2}}{2} - \frac{\tau k_B T}{2t} \tag{27}$$

It is instructive sheding light on Eq. (26) through the energy equipartition theorem, $(1/2)m_0\overline{v^2} = (1/2)k_B T$ [14]. It corresponds to $t \sim \tau$ and so to the mass–energy equivalence, $\tilde{E} = m_0$. When the particle behaves as though it were free ($t \ll \tau$), the movement is ruled by BSR; from the first collision on ($t > \tau$), it enters the SR regime (see Fig. 3). So, distinguishing the two stages of motion is never allowed. This is in line with GU, where (measuring) a volume just meant some uncertainity source. Moreover, it further supports the (fractal) Brownian picture for quantum motion, already suggested within other theoretical frames [15–17]. Here, Brownian and quantum movements would stand for the same phenomenon, linked by GSL, but observed at different scales.

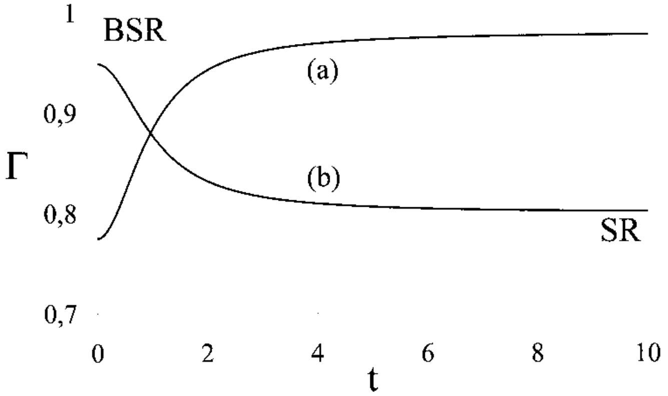

FIGURE 2 Behavior of the dual factor $\Gamma = \sqrt{(1-\gamma^2)}$ as a function of time, with $\kappa = D_c = 1$, (a) $v = 0.2$, $\delta = 0.6$ and (b) $v = 0.6$, $\delta = 0.9$.

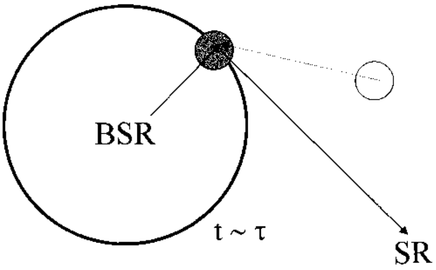

FIGURE 3 The BSR-to-SR motion transition for a particle embedded in a liquid.

SIMPLE LIQUIDS

Taking advantage of GSL enables one to attempt a heuristic derivation of the total correlation function of a liquid, $h = h(r)$. For fluids whose molecules interact with central additive forces, it provides directly the thermodynamic properties through the use of the state equation and the total energy expressions [18]. It also plays a remarkable role in investigating the amorphous state of matter in general (stability ratio in colloidal systems [19], suspension viscosity [20], etc.).

In terms of the total correlation function, we set the left side of Eq. (12) to:

$$\phi_\Lambda(r) = 4\pi\bar{\rho}\int r'^2 h(r')\,dr' \tag{28}$$

$\bar{\rho}$ being the macroscopic density. Consider now some atomic eigenstate (ψ), so that:

$$\phi_\lambda(r_0) = \langle\psi|\Psi(r_0')|\psi\rangle \tag{29}$$

angular and radial coordinates, as usual, being separated by $\psi(\overrightarrow{r_0}) = \psi_{\theta\varphi}(\Omega)\Psi(r_0)$. In this qualitative way, GSL takes the form:

$$h(\alpha r) \propto \Psi(r) \tag{30}$$

and can be used to describe the first correlation peaks in a liquid medium. Precisely, we start from pairwise distributions:

$$g(r^*) = 1 + h(r^*) \tag{31}$$

and the typical behavior of hydrogenoid waves [21]:

$$R_{n0}(\widetilde{r_n}) = N_{n0}e^{-\widetilde{r_n}}L_n^1(2\widetilde{r_n}) \tag{32}$$

where $N_{n0} \equiv (2a^{-3/2}/nn!)$ and $r_0 = na\widetilde{r_n}$. Observe that r^* is the radial dimension in unit of correlation length, while $a = a_0/Z$ is the ratio between Bohr radius and atomic number. To work in compact form, the asymptotic expression of Laguerre polynomials can be employed [22]:

$$\pi^{1/2}L_n^m(x) = e^{x/2}x^{-(m/2)-(1/4)}n^{(m/2)-(1/4)}\cos\left(2\sqrt{nx} - \frac{m\pi}{2} - \frac{\pi}{4}\right) + O\left(n^{(m/2)-(3/4)}\right) \tag{33}$$

After approximating the factorial term in N_{n0} by the Stirling formula, and simple algebraical manipulations, one obtains:

$$g(r^*) \sim 1 + Q_n r^{*-3/2} \cos\left(z\sqrt{r^*} - \frac{3\pi}{2}\right) \qquad (34)$$

with $r_0 \propto r^*$, $Q_n \propto \pi^{-1/2} n^{-3/4} (e/n)^n$ and $z \propto 2(2Z)^{1/2}$. The last two coefficients should be regarded as simple best fit parameters, not important here. Equation (34) is certainly too simple to describe the radial density behavior accurately. Its trend, however, fairly agrees with typical experimental behaviors. As examples, Fig. 4 shows (a) liquid argon at 91.8 K [23] and (b) molten lead at 613 K [24]. The agreement would encourage improving GSL by a deeper mathematical analysis. For instance, taking density fluctuations into account and expanding Eq. (29) through the whole orthonormal eigenfunction basis. This is an interesting issue left for future work.

CONCLUSIONS

1. The main scaling laws in polymer solutions was recently derived from a relativistic theory of Brownian movement in liquids (BSR). Universality arises whenever the indistinguishable molecular disorder is ordered progressively. Polymer statistics turns so out being intimately joint to the time-dependence of the (diffusive) motion equation.
2. To focus statistics at different length scales, BSR was then translated into basic geometry (GSR). We introduced the concepts of scaling law and uncertainty relation for geometry (GSL and GU), which seem being of interest to any relevant spatial statistics.
3. To combine kinematics (order) and diffusion (disorder) into an unique picture of motion, an example of dual SR has been proposed. Dual LT give birth to a complex phenomenology, undetermined between order and disorder. The relativistic energy

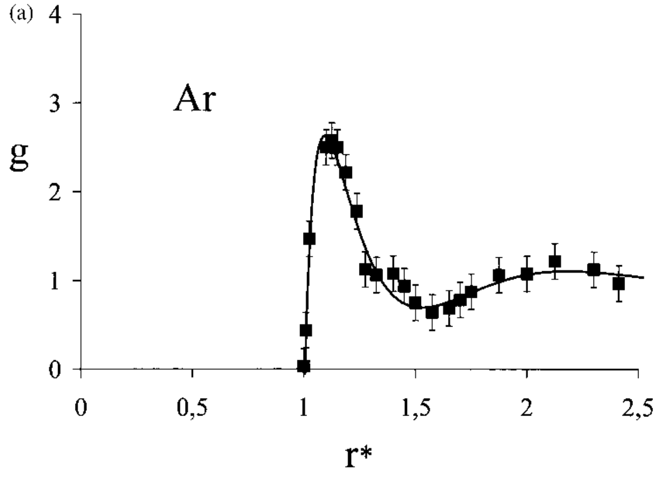

FIGURE 4 Pair correlation function of (a) liquid argon at 91.8 K and 1.8 atm ($Q_n \simeq 0.16, z \simeq 9.5$) and (b) molten lead at 613 K ($Q_n \simeq 0.2, z \simeq 9.2$).

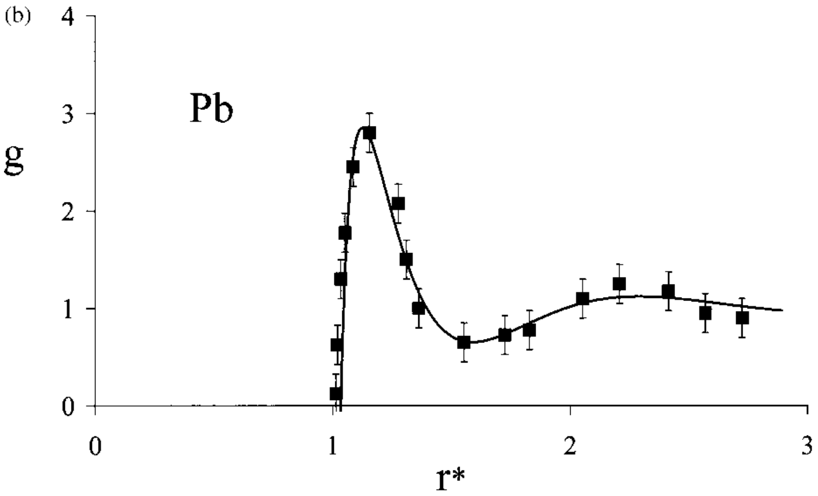

FIGURE 4 (Continued).

now depends on time, diffusivity and an impassible limit velocity, which refers to any particle either moving linearly or Brownianly. The energy–mass equivalence equals, in this framework, a notable result of statistical thermodynamics, the energy equipartition theorem.

4. In view of future analysis, it is lastly suggested to regard liquid density correlations as GSL from the atomic scale (i.e., wave functions), and to further investigate the nature of light, within a Brownian phenomenology.

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References

[1] S.A. Mezzasalma (2000). *J. Phys. Chem. B*, **104**, 4273.
[2] S.A. Mezzasalma, In: A. Hubbard (Ed.), *The Encyclopedia of Surface and Colloid Science*. Marcel Dekker Inc., New York (forthcoming).
[3] S.A. Mezzasalma (2001). *J. Stat. Phys.*, **102**, 1331.
[4] S. Weinberg (1972). *Gravitation and Cosmology*. John Wiley, New York.
[5] S.A. Mezzasalma (2002). *J. Colloid Interface Sci.*, **252**, 456.
[6] P.G. de Gennes (1979). *Scaling Concepts in Polymer Physics*. Cornell University Press, Ithaca.
[7] S.A. Mezzasalma (2000). *Chem. Phys. Lett.*, **326**, 237.
[8] S. Mezzasalma, work in progress.
[9] A. Messiah (1965). *Quantum Mechanics*, Vol. 1. North-Holland Publ., Amsterdam.
[10] L. Nottale (1997). *Astron. Astrophys.*, **327**, 867.
[11] J. Lyklema (1993). *Fundamentals of Interface and Colloid Science*. Academic Press, London.
[12] I. Prigogine and J. Géhéniau (1986). *Proc. Natl. Acad. Sci.*, **86**, 6245.
[13] F. Reif (1965). *Fundamentals of Statistical and Thermal Physics*. McGraw-Hill, New York.
[14] K. Huang (1987). *Statistical Mechanics*. John Wiley, New York.
[15] R.P. Feynman (1948). *Rev. Mod. Phys.*, **20**, 367.
[16] L.F. Abbott and M.B. Wise (1981). *Am. J. Phys.*, **49**, 37.
[17] G.N. Ord (1983). *J. Phys. A*, **16**, 1869.

[18] J.-P. Hansen and I.R. McDonald (1986). *Theory of Simple Liquids*. Academic Press, London.
[19] J. Jun, L. Blum and W. Haoping (1995). *Colloid Polym. Sci.*, **273**, 175.
[20] S.A. Mezzasalma (2001). *Colloid Polym. Sci.*, **279**, 22.
[21] L.D. Landau and E.M. Lifsits (1975). *Kvantovaja Mechanica*. Nauka, Moscow.
[22] I.S. Gradshteyn and I.M. Ryzhik (1973). *Tables of Integrals, Series and Products*. Academic Press, New York.
[23] A. Eisenstein and N.S. Gingrich (1942). *Phys. Rev.*, **62**, 261.
[24] D. Henderson (1971). *Physical Chemistry, An Advanced Treatise*, Vol. VIII-A. Academic Press, New York.
[25] We could also make the choice, $\omega = \delta^{-1}$, which equals to detect the time dilation law from the liquid frame of reference. Here, in Eq. (25), we set both TDLC from the same observer.

APPENDIX I

Consider the direct BLT, written as (prime denotes the chain frame of reference):

$$\begin{cases} \overline{r'^2} = \alpha\overline{r^2} + \beta t \\ t' = \gamma\overline{r^2} + \omega t \end{cases} \tag{35}$$

To determine their coefficients requires imposing two conditions, i.e., the interval invariance:

$$d\sigma'^2 = Ddt' - d\overline{r'^2} = d\sigma^2 \tag{36}$$

and the laws of time dilation and length contraction, both observed from any frame of reference that is linked to the chain, $\omega - (\gamma\beta/\alpha) \equiv \delta^{-1}$ [25]. Thereby:

$$\begin{cases} t' = (2-\delta)t + D^{-1}(1-\delta)\overline{r^2} \\ \overline{r'^2} = \delta\overline{r^2} + D(\delta - 1)t \end{cases} \tag{37}$$

and:

$$L_+ = \begin{pmatrix} \delta & -D(1-\delta) \\ D^{-1}(1-\delta) & 2-\delta \end{pmatrix} \tag{38}$$

To get the inverse BLT follows from setting $\delta \to \delta^{-1}$ into the former coefficients. We remember again that BLT are not invertible, unless $\delta = 1$. However, when equilibrium is reached, the new liquid medium is supposed giving rise to a new limit state ($\delta \to 1$) that, in the overall medium (liquid + chain), would prevent the two LT from colliding to each other.

APPENDIX II

The new elongational Lorentz factor satisfies the following limits:

$$(1-\gamma_L^2) \to \begin{cases} 1 - v^2 & t \to \infty \vee x_c \to 0 \\ \delta & t \to 0 \vee x_c \to \infty \\ \delta & v^2 \to 1 - \delta \\ 0 & v^2 \to 1 + \dfrac{\delta x_c}{t} \\ 1 - v^2 + \delta & (t \wedge x_c) \to \infty \\ \infty & t \to -x_c \end{cases} \tag{39}$$

and so, first terms in the MacLaurin expansions (from top, for v, t, x_{c}, δ) are:

$$\left(\frac{d\rho}{d\rho'}\right) \sim \begin{cases} \left(\dfrac{t+x_c\delta}{t+x_c}\right)^{1/2} \dfrac{\delta^2 x_c^2 + \delta x_c t(2-(v^2/2)) + t^2(1-(v^2/2)-(v^4/8)) + \cdots}{(t+\delta x_c)^2} \\[2ex] \delta^{1/2} + \dfrac{1-v^2-\delta}{2x_c}\delta^{-1/2} t + \cdots \\[2ex] (1-v^2)^{1/2} + \dfrac{v^2+\delta-1}{2t}(1-v^2)^{-1/2} x_c + \cdots \\[2ex] 1 + \dfrac{x_c}{2t}(1-v^2)^{-1/2}\delta + \cdots \end{cases} \tag{40}$$

Timelike reference frames, reading:

$$(1-v^2)t^2 + \delta x_c t = t'^2 + x_c t' \tag{41}$$

or else:

$$(1-v^2)t + \frac{\delta x_c}{2} = \pm\left\{\left(\frac{\delta x_c}{2}\right)^2 + (1-v^2)(t'+x_c)t'\right\}^{1/2} \tag{42}$$

yield:

$$\pm\, t \to \begin{cases} (1-v^2)^{-1/2} t' & x_c \to 0 \\ \delta^{-1} t' & x_c \to \infty \\ (1-v^2)^{-1/2}(t'^2 + x_c t')^{1/2} & \delta \to 0 \\ \delta^{-1} t'(x_c^{-1} t' + 1) & v \to 1 \\ t' & v \to 0 \wedge \delta \to 1 \\ \infty & v \to 1 \wedge \delta \to 0 \\ 0 & t \to -x_c \end{cases} \tag{43}$$

and (from top, v, t', x_{c}, δ):

$$|t| \sim \begin{cases} \dfrac{2(v^2+2)(t'+x_c)t' + \delta^2 x_c^2(1+v^2) - \delta x_c(1+v^2)(\delta^2 x_c^2 + 4x_c t' + 4t'^2)^{1/2} \ldots}{2(\delta^2 x_c^2 + 4x_c t' + 4t'^2)^{1/2}} \\[2ex] \dfrac{\delta x_c(\operatorname{sgn}(x_c)\delta x_c - 1)}{2(1-v^2)} + \dfrac{\operatorname{sgn}(x_c)}{\delta} t' + \cdots \\[2ex] \dfrac{|t'|}{(1-v^2)^{(1/2)}} + \dfrac{(1-v^2)\operatorname{sgn}(t') - \delta(1-v^2)^{(1/2)}}{2(1-v^2)^{(3/2)}} x_c + \cdots \\[2ex] \left\{\dfrac{t'(t'+x_c)}{1-v^2}\right\}^{(1/2)} - \dfrac{x_c}{2(1-v^2)}\delta + \cdots \end{cases} \tag{44}$$

For example, it can be seen that the first equation from t returns the usual time dilation law of SR, of course upon $x_c \to 0$.

APPENDIX III

Let $\Gamma = (1-\gamma^2)^{-1/2}$, with:

$$\gamma^2 = \frac{v^2 t + \chi D_c}{\kappa^2 t + D_c} \tag{45}$$

It can be expanded in the neighborhood of $(v, \delta) = (0, 0)$:

$$\begin{aligned}\Gamma(v,\delta) &= \Gamma(0,0) + \left(\frac{\partial\Gamma}{\partial v}\right)v + \left(\frac{\partial\Gamma}{\partial\delta}\right)\delta \\ &\quad + \frac{1}{2}\left\{\left(\frac{\partial^2\Gamma}{\partial v^2}\right)v^2 + 2\left(\frac{\partial^2\Gamma}{\partial v\partial\delta}\right)\delta v + \left(\frac{\partial^2\Gamma}{\partial\delta^2}\right)\delta^2\right\} \\ &\quad + \cdots\end{aligned} \tag{46}$$

where:

$$\begin{cases} \Gamma(0,0) = \left(1+\dfrac{D_c}{\kappa^2 t}\right)^{-1/2} \\ \left(\dfrac{\partial\Gamma}{\partial v}\right) = 0 \\ \left(\dfrac{\partial\Gamma}{\partial\delta}\right) = -\dfrac{1}{2}\dfrac{D_c}{\kappa^2 t}\left(1+\dfrac{D_c}{\kappa^2 t}\right)^{-1/2} \\ \left(\dfrac{\partial^2\Gamma}{\partial v^2}\right) = \dfrac{1}{\kappa^2}\left(1+\dfrac{D_c}{\kappa^2 t}\right)^{-1/2} \\ \left(\dfrac{\partial^2\Gamma}{\partial v\partial\delta}\right) = 0 \\ \left(\dfrac{\partial^2\Gamma}{\partial\delta^2}\right) = \dfrac{3}{4}\left(\dfrac{D_c}{\kappa^2 t}\right)^2\left(1+\dfrac{D_c}{\kappa^2 t}\right)^{-1/2} \end{cases} \tag{47}$$

The relativistic correction to the particle energy depends, accordingly, on diffusivity:

$$E = \left(\frac{\ell}{\kappa}\right)^2\left(1+\frac{D_c}{\kappa^2 t}\right)^{-1/2}\left(m_0\kappa^2 + \frac{m_0 v^2}{2} - \frac{m_0 D}{2t}\right) + O(v^4, \delta^2) \tag{48}$$